

## Research Article

## Design For Reliability With Weibull Analysis For Photovoltaic Modules

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### Abstract

The life data analysis is an important piece of the pie, but performing just the analysis is not enough to achieve reliable products. Rather, there are a variety of activities involved in an effective reliability program and in arriving at reliable products. In this paper two sets of photovoltaic modules were tested for the estimation of reliability. Whether either design meets the desired reliability goal. A comparison of sample averages using a student's *t* test reveals no statistical difference between the average cycles for Designs. But as a simple measure of central tendency, the sample average gives no information about the spread or shape of the distribution of failure times. Could the two designs' averages be the same, but their reliability be quite different? Reliability must be designed into products and processes using the best available science-based methods. Reliability practices must begin early in the design process and must be well integrated into the overall product development cycle. Design For Reliability (DFR) approach is suitable that describes the entire set of tools that support product and process design (typically from early in the concept stage all the way through to product obsolescence) to ensure that customer expectations for reliability are fully met throughout the life of the product. So a Scientific approach is adopted for comparing the reliability of the two designs. For this the Weibull analysis is performed. By performing a simple linear regression, parameter estimates can be obtained that will enable to make inferences about the two designs reliability.

**Keywords:** Weibull Statistics, Scale parameter, Shape parameter, Failure rate, Reliability function, Weibull cumulative distribution function.

### 1. Introduction

A PV module is comprised of dozens of materials and components, such as: back contact sheet, semiconductor, transparent conductors, encapsulates/sealants, cell-interconnects/solder joints, glass, and bypass diodes. Each module component, sub-component and/or underlying material has various and sometimes independent failure mechanisms that could lead to poor module reliability. Environmental stresses that can cause problems or exacerbate manufacturing defects include the following: Rapid thermal cycling due to sun and passing clouds; Extreme temperatures in winter and summer; Mechanical loading caused by wind and snow; and Moisture infiltration due to high humidity. These conditions can result in thermal expansion cracks, material warping and discoloration, conductor corrosion and contamination. Lifetime reliability prediction leverages expertise from multiple fields and requires complex experimental setups. Accelerated life testing, PV module qualification testing, and historical field data all play an important role in developing accurate predictions (A.K. Govil et al, 1983). Since it is too costly and impractical to collect field data for 25 years, accelerated test methods (Allis, et al, 1990) are often used to predict module performance.

Weibull analysis is a method for modeling data sets containing values greater than zero, such as failure data. Weibull analysis can make predictions about a product's life, compare the reliability of competing product designs, statistically establish warranty policies or proactively manage spare parts inventories, to name just a few common industrial applications. In fact, Weibull analysis techniques have become almost synonymous with reliability and achieving high reliability. The reality, though, is that although life data analysis is an important piece of the pie, performing just this type of analysis is not enough to achieve reliable products. Rather, there are a variety of activities involved in an effective reliability program and in arriving at reliable products. Achieving the organization's reliability goals requires strategic vision, proper planning, sufficient organizational resource allocation and the integration and institutionalization of reliability practices into development projects.

Three important statements summarize the best practice reliability philosophy of successful companies:

- 1) Reliability must be designed into products and processes using the best available science-based methods.
- 2) Knowing how to calculate reliability is important, but knowing how to achieve reliability is equally, if not more, important.

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3) Reliability practices must begin early in the design process and must be well integrated into the overall product development cycle. With such increasing complexity in all aspects of product development, it becomes a necessity to have a well-defined process for incorporating reliability activities into the design cycle (<http://www.reliasoft.com>).

Design for Reliability is a process specifically geared toward achieving high long-term reliability. This process attempts to identify and prevent design issues early in the development phase, instead of having these issues found in the hands of the customer.

**2. WEIBULL distribution**

Solar modules follow the ‘bath tub’ curve when they are being discussed under the context of failure and reliability analysis. In a more generalized principle, the modules experience 3 stages of behavior during their passage in time(Thibaut et al,2011). Stage 1 refers the ‘infant mortality’ cases where the initial failures are usually being high and decreases over time. Stage 2 refers the ‘normal life period’, the stage where the defects rate usually stays constant while the final stage refers the ‘wear out period’ which usually happens after 25 years at the end of life of solar modules provided they are operated in a way that they should be. Having understood the facts that the behavior is similar to other products, it is also not difficult to realize that the similar kind of statistics could be applied to predict the failure rate over time as in the case of products in any other industry. Utilizing this concept, the Weibull Statistics could be used to predict the failure rate over time using real failure returns data from the field. The Weibull distribution is one of the most widely used lifetime distributions in reliability engineering. It is a versatile distribution that can take on the characteristics of other distributions commonly known, based on the value of shape parameter (β) which is also known as slope parameter (as β is the slope in the line regression chart of Ln(Age) Vs Ln [ Ln(R(t)) ] ). In more generalized cases, a two parametric Weibull Statistic based on the Scale parameter (η) and Shape parameter (β) is appropriate enough, omitting the location parameter (γ). The probability density function then takes the form

$$f(t) = \frac{\beta}{\eta} \left(\frac{t-\gamma}{\eta}\right)^{\beta-1} e^{-\left(\frac{t-\gamma}{\eta}\right)^\beta}$$

- Equation (1)

The Weibull Reliability function could be derived from the above and is defined as

$$R(t) = e^{-\left(\frac{t-\gamma}{\eta}\right)^\beta}$$

- Equation (2)

The Weibull Failure rate function λ(T) is the ratio of Probability Density Function f(T) to Reliability Function R(T) and is given as(<http://www.weibull.com>)

$$\lambda(t) = \frac{f(t)}{R(t)} = \frac{\beta}{\eta} \left(\frac{t-\gamma}{\eta}\right)^{\beta-1}$$

- Equation (3)

Thus, the above equation requires the parameters β and η to be determined for the estimation of failure rate over time (<http://www.weibull.com>).

With some effort, the Weibull cumulative distribution function can be transformed so that it appears in the familiar form of a straight line: Y=mX+b:

$$F(x) = 1 - e^{-\left(\frac{x}{\alpha}\right)^\beta}$$

$$1 - F(x) = e^{-\left(\frac{x}{\alpha}\right)^\beta}$$

$$\ln(1 - F(x)) = -\left(\frac{x}{\alpha}\right)^\beta$$

$$\ln\left(\frac{1}{1 - F(x)}\right) = \left(\frac{x}{\alpha}\right)^\beta$$

$$\ln\left[\ln\left(\frac{1}{1 - F(x)}\right)\right] = \beta \ln\left(\frac{x}{\alpha}\right)$$

$$\ln\left[\ln\left(\frac{1}{1 - F(x)}\right)\right] = \beta \ln x - \beta \ln \alpha$$

Comparing this equation with the simple equation for a line, it is clear that the left side of the equation corresponds to Y, ln x corresponds to X, β corresponds to m, and -β ln α corresponds to b. Thus, when performing the linear regression, the estimate for the Weibull β parameter comes directly from the slope of the line. The estimate for the α parameter must be calculated as follows

$$\alpha = e^{-\left[\frac{b}{\beta}\right]}$$

- Equation (4)

**3. Linear model based time to failure estimation**

For simplicity the product was tested under a single stress and at a single constant stress level(Alexander et al,2011). Further assumed that times-to-failure data have been obtained at this stress level. The result of a single constant stress accelerated life test is considered for the reliability comparison of the two designed products. The times-to-failure at this stress level can then be easily analyzed using an underlying life distribution. A pdf of the times-to-failure of the product can be obtained at that single stress level using traditional approaches. This overstress pdf, can be used to make predictions and estimates of life measures of interest at that particular stress level. The objective in an accelerated life test, however, is not to obtain predictions and estimates at the particular elevated stress level at which the units were tested, but to obtain these measures at another stress level, the use stress level. To accomplish this objective, a linear model is assumed to traverse the path from the overstress pdf to extrapolate a use level pdf. The model or function can be described mathematically and can be as simple as the equation for a line.

Even when a model is assumed (i.e., linear, exponential, etc.), the mapping possibilities are still infinite since they depend on the parameters of the chosen model or relationship. For example, a simple linear model would generate different mappings for each slope value because we can draw an infinite number of lines through a point. If we tested specimens of our product at two different stress levels, we could begin to fit the model to the data. Obviously, the more points we have, the better off we are in correctly mapping this particular point, or fitting the model to our data.

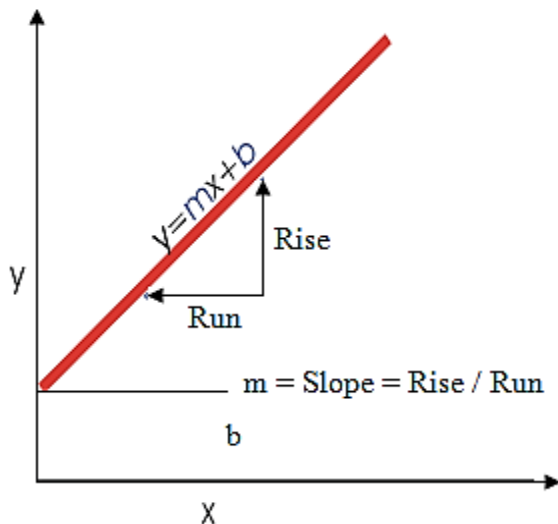


Figure 1 : Linear model for high stress and use stress for life

**4. Student t test with survival data**

In this paper two sets of photovoltaic panels were tested for the survival cycles. Whether either design meets the desired reliability goal. Unpaired t test results of the two sets of data ( Table 1) are given below(<http://www.statsoft.com>). P value and statistical significance are as follows. The two-tailed P value equals 0.6265. By conventional criteria, this difference is considered to be not statistically significant. The mean of Group One minus Group Two equals 4874.30 , 95% confidence interval of this difference: From -15810.85 to 25559.45 and intermediate values used are  $t = 0.4951$   $df = 18$ , standard error of difference = 9845.742.

A comparison of sample averages using a Student's t test reveals no statistical difference between the average cycles for designs (<http://www.weibull.com>). Could the two designs' averages be the same, but their reliability be quite different? Reliability must be designed into products and processes using the best available science-based methods. Reliability practices must begin early in the design process and must be well integrated into the overall product development cycle.

The desired reliability at 219000 hours ( 25 years of operation) is fixed as 0.90. This reliability goal is expressed mathematically as  $R(219000) = 0.90$ . Two sets of PV modules, each having ten modules labelled as “Test Module Set A” and “Test Module Test B” were considered

These 20 units were tested until they fails due to stress. Table 1

Table 1: Failure data from test sample sets.

PV Module Set 1			PV Module Set 2		
Test Module	Survival Hours	Years	Test Module	Survival Hours	Years
1	265000	30	11	234500	27
2	255000	29	12	265000	30
3	243500	28	13	240500	27
4	235055	27	14	235000	27
5	224588	26	15	234000	27
6	241050	28	16	250150	29
7	199000	23	17	199500	23
8	238000	27	18	221300	25
9	245000	28	19	258500	30
10	224500	26	20	183500	21

shows the number of cycles before failure for each item tested. The data in table 1 don't clearly indicate whether either module set meets the desired reliability goal. Both sets had at least one failure before 219000 hours, yet clearly the average number of cycles before failure exceeds 219000 hours for both designs.

Table 2: Failure data arranged in ascending order.

Test Module	Survival Hours	Years	Test Module	Survival Hours	Years
1	199000	30	11	183500	27
2	224500	29	12	199500	30
3	224588	28	13	221300	27
4	235055	27	14	234000	27
5	238000	26	15	234500	27
6	241050	28	16	235000	29
7	243500	23	17	240500	23
8	245000	27	18	250150	25
9	255000	28	19	258500	30
10	265000	26	20	265000	21

A comparison of sample averages using a student's t test reveals no statistical difference between the average hours for “Test Module Set A” and “Test Module Test B” (p-value = 0.965). But as a simple measure of central tendency, the sample average gives no information about the spread or shape of the distribution of failure times. Could the two averages be the same, but their reliability be quite different? So a more scientific comparison of the reliability can be proposed for the test module sets. The

student's t distribution is symmetric about zero, and its general shape is similar to that of the standard normal distribution. It is most commonly used in testing hypothesis about the mean of a particular population. The student's t distribution probability density function is defined as (for n = 1, 2, . . .):

$$f(t) = \frac{\Gamma(\frac{\nu+1}{2})}{\sqrt{\nu\pi} \Gamma(\frac{\nu}{2})} \left(1 + \frac{t^2}{\nu}\right)^{-\frac{\nu+1}{2}}$$

where  $\nu$  is the number of degrees of freedom and  $\Gamma$  is the Gamma function. This may also be written as

$$f(t) = \frac{1}{\sqrt{\nu} B\left(\frac{1}{2}, \frac{\nu}{2}\right)} \left(1 + \frac{t^2}{\nu}\right)^{-\frac{\nu+1}{2}}$$

where  $B$  is the Beta function.

**5.WEIBUL analysis with microsoft excel**

The Weibull shape parameter, called  $\beta$ , indicates whether the failure rate is increasing, constant or decreasing. A  $\beta < 1.0$  indicates that the product has a decreasing failure rate.

Table 3 : Preparation of test module set 1 for analysis

Survival Hours	Rank	Median Ranks	1/(1- Median Rank)	ln (ln (1/ (1- Median Rank)))	ln (Survival Hours)
A	B	C	D	E	F
199000	1	0.067308	1.072165	-2.66384	12.20106
224500	2	0.163462	1.195402	-1.72326	12.32163
224588	3	0.259615	1.350649	-1.20202	12.32202
235055	4	0.355769	1.552239	-0.82167	12.36757
238000	5	0.451923	1.824561	-0.5086	12.38003
241050	6	0.548077	2.212766	-0.23037	12.39276
243500	7	0.644231	2.810811	0.032925	12.40287
245000	8	0.740385	3.851852	0.299033	12.40901
255000	9	0.836538	6.117647	0.593977	12.44902
265000	10	0.932692	14.85714	0.992689	12.48749

This scenario is typical of "infant mortality" and indicates that the product is failing during its "burn-in" period. A  $\beta = 1.0$  indicates a constant failure rate. Frequently, components that have survived burn-in will subsequently exhibit a constant failure rate. A  $\beta > 1.0$  indicates an increasing failure rate. This is typical of products that are wearing out. Such is the case with both designs 1 and 2

have  $\beta$  values much higher than 1.0. The Weibull characteristic life, called  $\alpha$ , is a measure of the scale, or spread, in the distribution of data. It so happens that  $\alpha$  equals the number of hours or cycles at which 63.2 percent of the product has failed.

Table 4 : Weibull analysis result for modules test set 1

Weibull Analysis In Excel				
The Facts For A Weibull Plot			Data To Use For Excel Regression	
Rank	X-age to failure sorted	Y-plot position	Use This Data For X-axis	Use This Data For Y-axis
1	199000	0.067307692	-2.663843085	12.20106010
2	224500	0.163461538	-1.72326315	12.32163099
3	224588	0.259615385	-1.202023115	12.32202289
4	235055	0.355769231	-0.821666515	12.36757481
5	238000	0.451923077	-0.508595394	12.38002595
6	241050	0.548076923	-0.230365445	12.39275966
7	243500	0.644230769	0.032924962	12.40287222
8	245000	0.740384615	0.299032932	12.40901349
9	255000	0.836538462	0.593977217	12.44901882
10	265000	0.932692308	0.992688929	12.48748510
Regression Status				
			$\beta =$	14.410
			$\eta =$	245155.565
			$R^2 =$	0.9564

In other words, for a Weibull distribution  $R(\alpha = 0.368$ , regardless of the value of  $\beta$ . The distribution is

$$R(t) = e^{-\left(\frac{t}{\alpha}\right)^\beta}$$

where  $t$  is the time (or number of cycles) until failure.

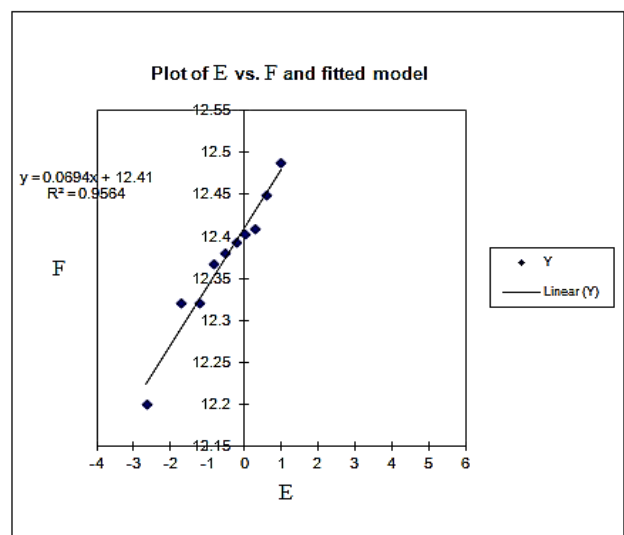


Figure 2 : ln (ln (1/ (1- Median Rank))) x ln ( Survival Hours ) for test module set 1.

Weibull and regression analysis can be done using the Microsoft Excel and the result are shown in table 4 and 6. The regression status is as follows. For PV module test set



1, the  $\beta = 14.410$  and the  $\alpha$  ( marked as  $\eta$  in table ) = 245155.565. For PV module test set 2, the  $\beta = 9.982$  and the  $\alpha$  ( marked as  $\eta$  in table ) = 243309.680.

Table 5: Preparation of test module set 2 for Weibull analysis.

Survival Hours	Rank	Median Ranks	1/(1- Median Rank)	ln (ln (1/ (1- Median Rank)))	ln (Survival Hours)
A	B	C	D	E	F
183500	1	0.067307692	1.072164948	-2.6638	12.11996995
199500	2	0.163461538	1.195402299	-1.7233	12.20356952
221300	3	0.259615385	1.350649351	-1.202	12.30727453
234000	4	0.355769231	1.552238806	-0.8217	12.36307639
234500	5	0.451923077	1.824561404	-0.5086	12.36521087
235000	6	0.548076923	2.212765957	-0.2304	12.36734079
240500	7	0.644230769	2.810810811	0.03292	12.39047537
250150	8	0.740384615	3.851851852	0.29903	12.42981602
258500	9	0.836538462	6.117647059	0.59398	12.46265097
265000	10	0.932692308	14.85714286	0.99269	12.4874851

Table 6 : Weibull analysis result for modules test set 2

Weibull Analysis In Excel				
The Facts For A Weibull Plot			Data To Use For Excel Regression	
Rank	X-age to failure sorted	Y-plot position	Use This Data For X-axis	Use This Data For Y-axis
1	183500	0.067307692	-2.663843085	12.11996995
2	199500	0.163461538	-1.72326315	12.20356952
3	221300	0.259615385	-1.202023115	12.30727453
4	234000	0.355769231	-0.821666515	12.36307639
5	234500	0.451923077	-0.508595394	12.36521087
6	235000	0.548076923	-0.230365445	12.36734079
7	240500	0.644230769	0.032924962	12.39047537
8	250150	0.740384615	0.299032932	12.42981602
9	258500	0.836538462	0.593977217	12.46265097
10	265000	0.932692308	0.992688929	12.4874851
Regression Status				
	$\beta =$	9.982		
	$\eta =$	243309.680		
	$R^2 =$	0.9640		

6. Result and interpretation

The Weibull shape parameter, called  $\beta$ , indicates whether the failure rate is increasing, constant or decreasing. A  $\beta < 1.0$  indicates that the product has a decreasing failure rate. This scenario is typical of "infant mortality" and indicates that the product is failing during its "burn-in" period. A  $\beta = 1.0$  indicates a constant failure rate.

Frequently, components that have survived burn-in will subsequently exhibit a constant failure rate. A  $\beta > 1.0$  indicates an increasing failure rate. This is typical of products that are wearing out. Such is the case with the two sets of PV modules. Both PV module set 1 and 2 have values 14.410 and 9.982 respectively, which are much higher than 1.0. The PV modules fail .i.e. they wear out(<http://rac.alionscience.com>).

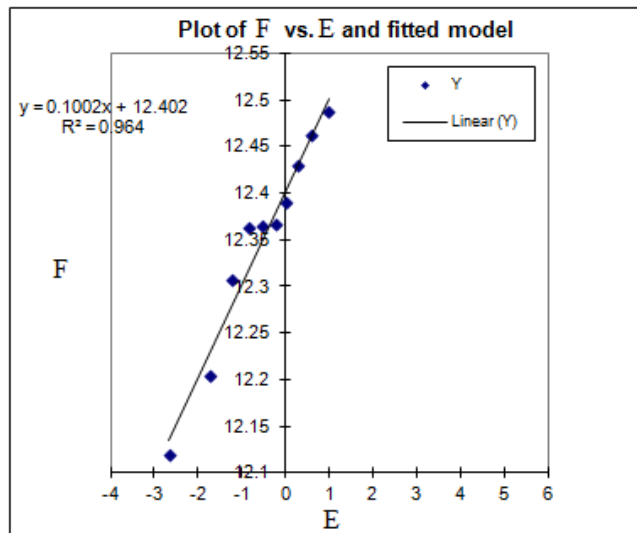


Figure 3 : ln (ln( 1/( 1- Median Rank))) x ln ( Survival Hours ) for test module set 2.

The Weibull characteristic life, called  $\alpha$  (  $\eta$  in the regression analysis ) , is a measure of the scale, or spread, in the distribution of data. It so happens that  $\alpha$  equals the number of hours or cycles at which 63.2 percent of the product has failed. In other words, for a Weibull distribution  $R(\alpha=0.368)$ , regardless of the value of  $\beta$ . From the PV module set 1 about 37 percent modules, i.e. maximum 4 modules should survive at least 245155 hours ( 28 Years). From the PV module set 2 about 37 percent modules should survive at least 243310 hours ( 27.7 Years).

It is not revealing whether either PV module set 1 or two meets the reliability goal of  $R(219000) \geq 0.90$ . For this, a Weibull distribution is assumed and hence using the formula:

$$R(t) = e^{-\left(\frac{t}{\alpha}\right)^\beta}$$

$$R(219000) = e^{-\left(\frac{219000}{245155}\right)^{14.41}} = 0.8214$$

$$R(219000) = e^{-\left(\frac{219000}{243310}\right)^{9.982}} = 0.7049$$

The calculation shows that both modules set have less than 90% reliability at 25 years, 82.14% and 70.49% respectively. But practically it will be less due to many environmental factors which are not accounted in the analysis. Figures 2 and 3 are the result of the analysis based on the real life table data that is fitted on the straight

line using the line regression method. The predicted straight line is the best fit to the observed values and represents the maximum likelihood for parameters  
 Table 7 : Calculated Reliability with Weibull parameters.

Time in Years	Reliability in %	
	Test Set 1	Test Set 2
15.0	99.98	99.78
20.0	99.21	96.30
22.0	96.92	90.70
23.0	94.25	85.89
23.5	92.25	82.81
24.0	89.65	79.24
25.0	82.14	70.49

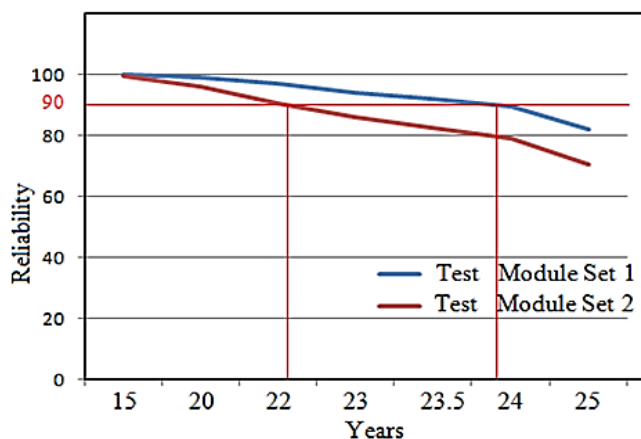


Figure 4 : Year versus Reliability curve for the test modules.

estimation. In this analysis, for the PV module set 1, the  $\beta$  resulted is 14.410 which is much larger than 1 and represents 'wear out' case. The other parameter that required is the Scale parameter ( $\eta$ ), is the measure of spread or scale in the distribution of data. The estimation of parameters  $\beta$  and  $\eta$  opens many possibilities not only in predicting the failure rate over time but also to estimate the likely cumulative defective rate at modules warranty expiry line.

### 7. Conclusion

According to the IEC standard tests, all the failure modes that happen with the solar modules in the field in were classified in 8 failure modes related to interconnects, solder bonds, encapsulate lamination, metallization corrosion, cell crack or break, glass break and hot spots. The quality tools such as FMEA and Pareto are useful in identifying the vital modes, capturing the maximum occurrence of the defects but do not provide information on the failure modes behavior with time. The failure modes that are explained by only their maximum occurrences may not be adequate in this case for suitable containment actions. Extending the above discussed analysis for studying, each failure modes individually, their behavior with time would strengthen the analysis further and lead us to take the right actions. The cumulative failure rate of solar modules at their end-of-lives could be predicted using Weibull statistics and a Kaplan-Meier life table could be used to estimate the maximum likelihood of required parameters  $\beta$  and  $\eta$ . The cumulative failure rate after 25 years is expected below the industry acceptable levels. It is also concluded that similar kind of analysis could be extended to cover each defects in depth for a suitable course of containment actions.

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